at the actual throat, whose location is governed by the wedge angle of the trailing edge, a clear dependence on this angle must have existed.

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Stabilization of Supersonic Combustion by a Free Recirculating Bubble: A Numerical Study

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I. Introduction

DURING the past few years, the problem of ignition and stabilization of combustion in supersonic flows has been addressed by various authors, from both experimental[1] and numerical[2] points of view. An issue of fundamental importance is the stabilization of combustion when the vehicle flight Mach number is low. In this case, the flow Mach number and static temperature within the combustion chamber are too low to allow self-ignition in mixing layers and jets, and stable oblique detonation regimes are difficult to obtain. One way to circumvent this shortcoming, previously suggested by Winterfeld[3] and by Nedungadi and Lewis,[4] is the stabilization of combustion within a free recirculating bubble. The problem of a free recirculation bubble in supersonic nonreactive flows has been previously studied by various authors.[4–10] In these works, a recirculation bubble is generated by the interaction of an oblique shock wave and an incoming flow containing a finite region where a total pressure deficit exists. This total pressure deficit region is, in the aforementioned papers, a region of swirling flow, generated either by a wing-like structure or an injector. Depending on the relative intensities of the swirl and of the shock wave, it is possible to obtain either a small bulging of the shock or a large deformation of the shock wave, the latter leading to the formation of a free recirculating bubble. More recently, Mahesh[10] has shown that the bubble can be obtained without any swirl, the controlling parameter in the formation of the free recirculating bubble being the velocity deficit.

In the present work, the use of a recirculation bubble is proposed to achieve full premixing, ignition, and stabilized combustion in supersonic flows. A first description of the flowfield obtained when combustion occurs is given.

II. Mathematical Formulation of the Problem and Numerical Procedure

In this Note, the two-dimensional unsteady conservation equations of mass, momentum, energy, and species mass fraction are solved for a multicomponent gas mixture[2] in the geometry shown in Fig. 1 of a uniform airflow surrounding a methane axisymmetric jet. The turbulent transport model used is a gradient closure, with a mixing length turbulent viscosity $\nu_t = C_\delta l^2 |\partial u/\partial y|$, where $C_\delta = 0.008$ and l is the mixing length, assumed to be equal to the scale δy of the gradient, and δU is taken as the maximum difference of velocity in the incoming flow, $400 \leq \delta U \leq 700$ m/s. In fact, this leads to what can be considered an overestimation of the turbulent viscosity $\nu_t = 4000$ cm^2/s. This relatively large value of ν_t can be justified by the existence of large-scale fluctuations in the flow external to the configuration studied. Computations showing the exact influence of the turbulent viscosity are currently in progress. The reaction rate of methane and air is calculated assuming a global chemical kinetics model[11] and using the average temperature and mass fractions. This global modeling of the chemical process is known to not be very accurate for computing the induction time for a large range of temperature and pressures. Even if the drastic modeling of both turbulent transports and chemistry used is not suited to predict in detail the local characteristics of the flow, it is deemed sufficient to describe the gross features of the phenomena studied in this Note.

The numerical procedure used here is based on the Liou[12] advection upstream splitting method, with a second-order MUSCL extrapolation and MINMOD limiter for the primitive variables (ρ, u, v, p, Y_k). Details concerning the solution procedure can be found in Ref. 2.

III. Results and Discussion

The interaction of a swirling flow with an oblique shock wave may lead to complex three-dimensional phenomena.[4–9] To retain only the most significant aspects of such an interaction, we treat this problem here in a two-dimensional axisymmetric form. In a way that is similar to that of the work of Metwally et al.[8] and Mahesh,[10] but using a different flow/shock wave interaction, we obtain free recirculating bubbles without swirl when the velocity of the core is smaller than the external flow velocity.

As shown schematically in Fig. 1, a central stream of partially premixed fuel (stoichiometric methane/air mixture, denoted by the subscript c) that is surrounded by an airstream (denoted by the subscript ∞) is considered. Both streams have the same static pressure $p_\infty = p_c$ and, for the sake of simplicity, total temperature $T_{t\infty} = T_{tc}$ but different velocities, i.e., u_∞ and u_c. At $x = 0$ both

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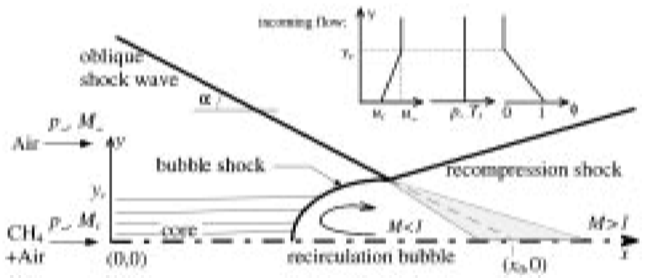

Fig. 1 Schematic representation of the strong interaction between an oblique shock wave and a core flow where a deficit in total pressure exists.

the equivalence ratio and the longitudinal velocity u vary linearly with the radial coordinate y, $u = u_c - \Delta U(y/y_c)$; for $y \leq y_c$, $\Delta U = u_c - u_\infty$, and for $y > y_c$, $u = u_\infty$. Such a choice of inflow conditions result in a transverse total pressure gradient. The shock wave that interacts with these two streams is produced by an outer airstream that has a higher static pressure. This oblique shock is characterized by its angle α and hypothetical crossing point along the symmetry line x_0. The flow conditions are given in the figure captions. In the present case of the numerical simulation of an axisymmetric flow, it seems more convenient to use this method to generate an oblique shock wave, because the flow deflection by a surrounding wall would generate a shock with nonconstant angle. All of the computations discussed next are mesh- and time-independent results, unless stated otherwise.

A. Formation of the Bubble; Nonreactive Case

Figure 2, where the fields of density are plotted for four different core Mach numbers M_c (and thus of $\Delta U/u_\infty$) and for identical airflows ($M_\infty = 3$), shows the existence of a critical velocity deficit for the onset of the recirculation bubble. The case where $M_c = 3$, i.e., for $\Delta U/u_\infty = 0$, is close to the limit of a regular reflection of the initial oblique shock wave: an increase of only 2 deg leads to a Mach reflection. The result obtained for $M_c = 1.5$ corresponds to a case of a weak interaction, first described by Metwally et al.[8]: the resulting flowfield contains a curved shock followed by a subsonic region without any recirculation bubble. The case of intermediate core Mach number $M_c = 1.3$ corresponds to a strong interaction[8]: the recirculation bubble is fully developed, and the shock deformation is much larger than in the case where $M_c = 1.5$. The lowest M_c case, i.e., $M_c = 1.05$, results in an upstream propagation of the bubble that may be influenced by the choice of the conditions at the downstream boundary, where the flow is now subsonic.

For these initial conditions, a small velocity deficit between the core and external flows is sufficient to obtain a well-formed Mach reflection, thus giving birth to a normal shock wave in the core region. This is clearly neither sufficient nor necessary to generate a recirculating bubble. The critical conditions for the existence of the bubble are dependent on the exact flow configuration, i.e., the initial shock strength, external flow Mach number, and core velocity deficit.

An analysis is currently being performed to determine precisely the conditions for which the onset of the recirculation bubble occurs. When M_∞ and α are fixed, as in the present Note, obtaining the recirculation bubble is found to be a function of the velocity deficit $\Delta U/u_\infty$ only.

For the airflow Mach number $M_\infty = 3$ and shock angle $\alpha = 55$ deg, the critical velocity deficit required to obtain numerically a recirculation bubble lies in the range $-0.3 < \Delta U/u_\infty < -0.25$, which corresponds to the conditions of Figs. 2b and 2c. Even though, as shown by Mahesh,[10] a critical value of $\Delta U/u_\infty$ will exist beyond which recirculation bubbles are obtained, a specific analysis is required to derive the exact critical conditions in the flow configuration studied because the shock wave configuration considered here is quite different from that assumed in the paper of Mahesh.[10]

B. Discussion of the Conditions Necessary to Obtain a Stable Solution; Reactive Case

For ignition of the gases to be obtained, the residence time τ_r of the mixture within the bubble must be larger than the characteristic chemical ignition time τ_c. In the case where $M_c = 1.3$, presented in the preceding section, a rough estimate of τ_r based on the inert dimension of the bubble and on a velocity of 90 m/s at its outer edge gives $\tau_r \approx 1$ ms. An induction time τ_c of the chemical kinetics smaller than 1 ms is required for self-ignition to occur. This is obtained for a stoichiometric methane/air mixture, following the overall chemical mechanism,[11] when the temperature within the recirculation bubble is higher than 1400 K. The present

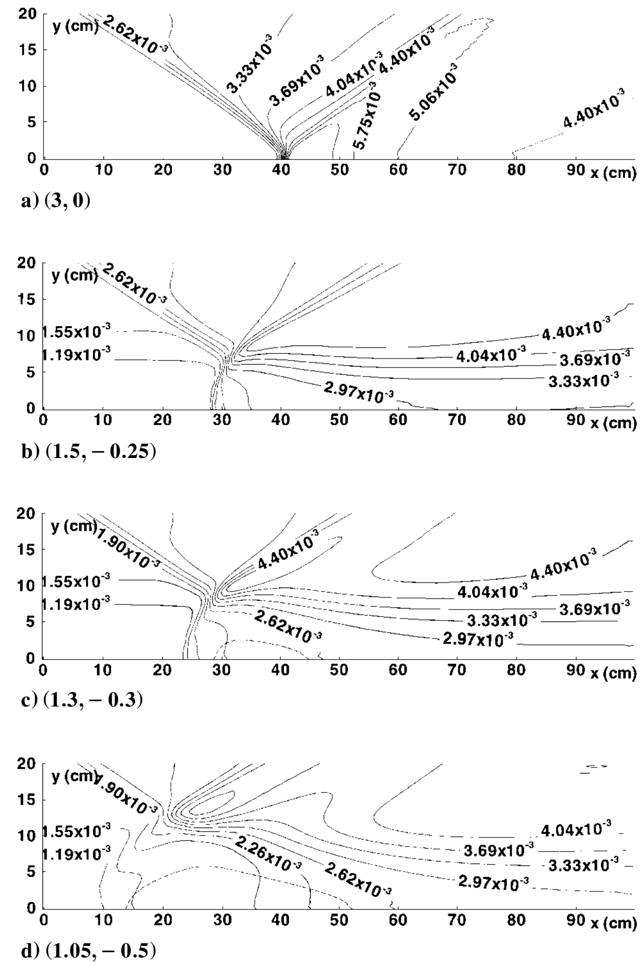

Fig. 2 Fields of density (g/cm^3) for different core Mach numbers for inert flow, (M_c, $\Delta U/u_\infty$): $M_\infty = 3$, $T_{t\infty} = T_{tc} = 1500$ K, $p_\infty = p_c = 0.26$ MPa, $y_c = 11$ cm, $\alpha = 55$ deg, and $x_0 = 40$ cm. The dashed line corresponds to $u = 0$. Mesh contains 125^2 uniformly distributed points.

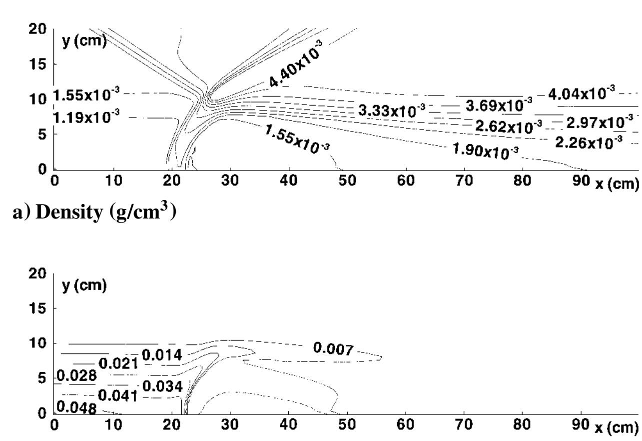

Fig. 3 Fields of density and methane mass fraction for reactive flow; $M_c = 1.3$, $M_\infty = 3$, $T_{t\infty} = T_{tc} = 1500$ K, $p_\infty = p_c = 0.26$ MPa, $y_c = 11$ cm, $\alpha = 55$ deg, and $x_0 = 40$ cm. The dashed line corresponds to $u = 0$. Mesh contains 250 × 125 uniformly distributed points.

computations show that this condition is fulfilled for $M_c \leq 1.3$. The fields of density and methane mass fraction obtained after ignition and stabilization of combustion are plotted in Fig. 3. It can be observed that combustion results in an increase in the size of the recirculation bubble and in an upstream displacement of the leading shock of about 7 cm. These features could be due to the decrease of the density and the dynamic turbulent viscosity $\mu_t = \rho \nu_t$ and thus to the corresponding change of the force balance acting on the bubble.

IV. Conclusion

In this work, a technique to stabilize combustion within supersonic flows using a free recirculating bubble has been presented. Although both the turbulence model and chemical kinetics mechanism used here remain extremely simple, the overall features of the flow are expected to stay unchanged when these approximations are refined for the sake of quantitative analysis. It is clear that the computations performed here are unable to predict unsteady properties of the system, such as vortex shedding or oscillations of the bubble, that can play a significant role in the ignition and stabilization capabilities of this system. Calculations to determine precisely the role of different flow parameters on the stability limits of the free recirculation bubble with combustion are now being performed. The preliminary results presented here need to be extended to the case of practical interest of a swirling jet.

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Some Practical Complete Modal Spaces and Equivalence

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Introduction

THE modal expansion method has been found to be very useful for modal synthesis, calculation of eigenvector derivatives, model correction, and reduction of dynamic models. Because there exists numerical error due to truncation of modes, the precision of the results sometimes is poor. To reduce the error of truncated modes, a practical complete modal space (PCMS) is developed. This Note briefly describes successful applications of the PCMS in some practical engineering areas. To satisfy the requirement in different applications, the PCMS has been further improved.

In the PCMS method, some lower-order modes including rigid-body modes are obtained from solving the eigenequation. All higher-order modes are replaced by the equivalent higher-order modes that are given by using a simple matrix projection approach. This replacement means the entire contribution of the equivalent higher-order modes is equivalent to that of the original higher-order modes. Also the subspace spanned by the equivalent higher-order modes is directly equivalent to the subspace spanned by original higher-order modes. The direct proof of equivalence between two subspaces spanned by both the equivalent higher-order modes and original higher-order modes is discussed in this Note.

With respect to the modal synthesis, the existing free-interface method,[1] the fixed-interface method,[2] and the mixed-interface method[3] are all approximately substructural coupling methods based on an incomplete modal space. The accurate modal synthesis methods[4–6] based on the PCMS method can give better precision of any order of modes for assembly structure because the precision of the Rize analysis is determined principally by the completeness of the basis of the vector space. In the calculation of eigenvector derivatives, Fox and Kapoor[7] developed an incomplete modal expansion technique. Wang[8] improved the Fox–Kapoor method by adding a static correction term to the modal expansion formula. But their methods merely guarantee that the precision of the eigenvector derivatives of a few lower-order modes is good. Otherwise a complete modal method[9] based on the PCMS theory can make the precision of the derivatives of many higher-order modes accurate. In regard to the reduction of a dynamic model, Kammer[10] proposed an incomplete modal method that can only guarantee that some lower-order modes of the reduced model are exact inside the frequency range of interest. Reference 11 makes available many modes of the reduced model outside the range of interest. In the field of model correction, Berman et al.,[12] Berman and Wei,[13] Zhang and Li,[14] and Kabe[15] developed various methods of model correction, which are based on an incomplete modal space. A complete mode-type method was described in Ref. 16. In the present method, the PCMS formed by lower-order measured modes and equivalent higher-order modes is arranged as the reference base. Based on this reference base, there are matrix-type, element-type, submatrix-type, and design-parameter-type methods. The results from the matrix-type and element-type methods based on the PCMS show that the

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